

INTERFACE OF GRAVITATIONAL AND QUANTUM REALMS

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The talk centers around the question: Can general-relativistic description of physical reality be considered complete? On the way I argue how – unknown to many a physicists, even today – the “forty orders of magnitude argument” against quantum gravity phenomenology was defeated more than a quarter of a century ago, and how we now stand at the possible verge of detecting a signal for the spacetime foam, and studying the gravitationally-modified wave particle duality using superconducting quantum interference devices.

Keywords: Quantum gravity phenomenology, gravitationally-induced phases, flavor-oscillation clocks, gravitationally-modified wave particle duality.

Preamble

The idea for the First IUCAA Meeting on the Interface of Gravitational and Quantum Realms (17-21 December 2001, Pune, India) arose during a walk, a year before, with Naresh Dadhich. A reader who was not at IGQR-I is likely to find a contradiction between what I write here and what Naresh Dadhich and I write in the opening lines of the Preface. For that reader I note that the physics walks at IUCAA often begin with a left turn exit from the main IUCAA entrance, they wind through a narrow road on the outskirts of an open field. Mid way in the walk, on the left of that narrow lane, is a tree. The tree provides shade for a *chai* (“tea”) and *samosa* break, and hosts birds of several species. After the *chai* the walk continues. The walk, punctuated by a *chai* and *samosa* break, is an important part of life at IUCAA. Such walks inspire a whole range of new ideas and provide an opportunity for monastic reflections.

1. Introduction

The foundations of the modern gravitational and quantum frameworks were established in an intellectually turbulent era of the early twentieth century. The rapid developments of the theory of general relativity on the one hand, and the similarly fast evolution of the theory of quantum mechanics on the other – under schools which were essentially opposed in their philosophical outlooks [1, 2] – have, in my opinion, left the interface of the two frameworks largely unexplored.

The Bell paper [3] finally placed the concerns on the incompleteness of quantum framework [1] to experimental front [4, 5, 6, 7, 8]. Yet, if it was asked for the quantum realm, “Can quantum-mechanical description of physical reality be considered complete?,” then asking, “Can general-relativistic description of physical reality be considered complete?,” may, I suspect, lead to interesting insights into the interface of the two rival views on physical reality.

Towards this end, in the next section I briefly review the canonical dimensional arguments which place the interface of the gravitational and quantum realms (IGQR) beyond the reach of any quantum gravity phenomenology (QGP) in an environment independent manner. However, since many aspects of the IGQR depend on the gravitational environment/context, the canonical wisdom is no longer applicable, and I argue for a viable QGP program for exploring IGQR. The remainder of the talk is devoted to theoretical remarks on the IGQR, and suggestions for its exploration in terrestrial laboratories.

2. Viability of QGP program for exploring IGQR: Defeating the 40 orders of magnitude argument.

To prepare for examining the experimental viability of exploring the IGQR, I first enumerate, for ready reference, *some** of the relevant numbers in Table 1. Using the listed constants, in an environment-independent context, one readily obtains various scales at which quantum-gravity shall become manifest

$$\lambda_P = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-33} \text{ cm}, \quad f_P = \frac{c}{\lambda_P} = 1.85 \times 10^{43} \text{ Hz}, \quad (1)$$

$$m_P = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-5} \text{ g}, \quad \gamma = \frac{G m_n^2 / \hbar c}{e^2 / \hbar c} = 8.1 \times 10^{-37}. \quad (2)$$

In the canonical arguments, the smallness of Planck length, λ_P , in comparison to atomic,[†] and nuclear dimensions, renders IGQR beyond the reach of terrestrial experiments. At the same time, in comparison to energies to which elementary particles can be accelerated in high-energy physics accelerators, the Planck mass, m_P , is seen to be exceedingly large to allow any terrestrial exploration of IGQR. The extreme smallness of γ , similarly, encodes the same conclusions. These arguments, owing to second of Eqs. (2), are known as the “40 orders of magnitude” disparity between realm of quantum electrodynamics and the regime of quantum gravity.

As we move through this talk, we shall see that such arguments are misleading. To make first of such arguments, in Table 2, I collect together some of the quantities that define the terrestrial gravitational environment. In conjunction with Table 1,

*That is, ignoring the relevant cosmological and astrophysical numbers.

[†]Typical atomic dimension is of the order of Ångström, Å = 10^{-8} cm. Typical nuclear length scale is of the order of a Fermi, $F = 10^{-13}$ cm.

Proton charge	e	4.80×10^{-10} esu
Gravitational constant	G	6.67×10^{-8} dyn-cm ² /g ²
Planck's constant/ 2π	\hbar	1.05×10^{-27} erg-s
Speed of light in vacuum	c	3×10^{10} cm/s
Nucleon mass	m_n	1.67×10^{-24} g

Table 1: For ready reference, some constants relevant for the interface of the gravitational and quantum realms. All numbers cited here, and rest of the paper, are in the sense of “ \approx .”

Earth mass	M_{\oplus}	5.98×10^{27} g
Earth radius [Mean]	R_{\oplus}	6.37×10^8 cm
Solar mass	M_{\odot}	2×10^{33} g
“Great Attractor region (GAR)” mass	M_{GAR}	$10^{17} M_{\odot}$
“GAR - Milky Way (MW)” distance	R_{GAR-MW}	150 million light years, i.e. 1.42×10^{26} cm

Table 2: For ready reference, some quantities associated with our gravitational environment.

it allows us to construct the following dimensionless gravitational potentials:

$$\phi_{\oplus} = -\frac{GM_{\oplus}}{c^2 R_{\oplus}} = -6.96 \times 10^{-10}, \quad \phi_n = -\frac{Gm_n}{c^2 F} = -1.24 \times 10^{-39}. \quad (3)$$

Immediately, I take note of the circumstance that in going from the nuclear realm to laboratories on the surface of the earth we gain thirty orders of magnitude (towards defeating γ):

$$\frac{\phi_{\oplus}}{\phi_n} = 0.56 \times 10^{30}, \quad (4)$$

In addition, if we experiment with thermal neutrons with about an Å wavelength, then for a 10 cm table-top interferometer arm, we obtain a dimensionless number, 10^9 . The interplay of these two large numbers comfortably overcomes the perceived disadvantage for QGP in IGQR.

In fact such arguments underlie at the heart of the gravitationally-induced phases *experimentally* observed in neutron [9, 10, 11, 12] and atomic [13] interferometry. I summarize the experimental results in Table 3. The cited experiments probe classical gravity in a quantum context and verify equality of the inertial and gravitational masses at different levels of accuracy. Note that experiments in Ref. [10, 14] refer to neutron interferometry, while the experiment of Ref. [13] is in the context of atomic interferometry. The discrepancy noted in [14], if it persists experimentally, would be a serious challenge to the equality of the inertial and gravitational masses for neutron and carries serious implications, not only for classical theory of gravitation, but also for quantum gravity. However, this circumstance

Year [Ref.]	Observation
1975 [10]	“First verification of the principle of equivalence in the quantum limit.” $m_i = mg$, at 1% level.
1997 [14]	$m_i \neq m_g$, a few part in 1000 discrepancy.
1999 [13]	“...best confirmation of equivalence principle between a quantum and macroscopic object.”
2000 [15]	Undertakes to study the discrepancy of Ref. [14]. No definitive conclusion, yet.

Table 3: Status of equivalence principle in the *quantum* realm.

is besides the point. The essential point to be made is that *experiment* must remain the main guide and any empirical confirmation/surprise carries implications for classical as well quantum aspects of gravity – as the latter is not immune to the structure of the former.

3. Equivalence principle in the IGQR

When one speaks of exploring equivalence principle one often has in mind macroscopic classical objects. Yet, as is clear from the brief discussion of the previous section, since 1975 the neutron interferometry has explored equivalence principle in the context of a single mass eigenstate. Since 1997, there is a statistically significant signal for a violation of equivalence principle (VEP). It is associated with the evolution of a neutron in the classical gravitational field of Earth. It has been discussed at some length in Ref. [16, 17].

However, more than a decade before neutron interferometry experiments were initiated, Schiff [18], and Morrison and Gold [19], considered the possible violations of equivalence principle in quantum contexts. Good [20] was perhaps the first to note of the interesting possibilities which *quantum systems in linear superposition of different mass/energy eigenstates* offer to explore IGQR. Further theoretical investigations of such interesting quantum test particles – with no classical counterpart – occurred in the context of neutrino oscillations [21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

First dedicated experiment which studied neutrino-like systems[‡] in the classical gravitational field of Earth was in the Stanford laboratory of Chu and his collaborators [13]. It verified the principle of equivalence to a few parts in 10^9 , and at the similar level of accuracy established that the specific quantum test particle experienced same gravitationally-induced acceleration as a classical macroscopic glass object (same acceleration to 7 parts in 10^9).

In all early gedanken experiments in the gravitational realm the attention was invariably confined to mass eigenstates. However, in the quantum realm – and now

[‡]Specifically, an atom in linear superposition of different energy eigenstates.

with mounting evidence for neutrino oscillations – one must in addition consider gedanken experiments which allow quantum test particles in linear superposition of different mass/energy eigenstates.[§] Now, if inertial and gravitational masses are considered operationally independent, then, for such test particles (which have no classical counterpart), the equality of inertial and gravitational masses cannot be claimed beyond certain fractional accuracy. This was first noted in Refs. [16, 17]. These fractional accuracies are determined by the underlying mass eigenstates, and the mixing matrix. As accuracy of the Stanford-like experiments improves, these flavor-dependent fractional accuracies will become accessible to experimental investigations.

References [21, 22, 23, 24, 25, 26, 27, 28] provide extensive discussion of flavor oscillation clocks – a term I coined a few years ago to emphasize unique nature of these systems to investigate gravity with quantum test particles ¶ – and the effect of different gravitational environments. Should a violation of equivalence principle exist, the essentially constant gravitational potential produced by cosmological fluctuations in matter density would become observable in *local* measurements of gravitationally-induced redshift of flavor oscillation clocks. Such gravitational potentials carry a typical dimensionless value given by (see, Great Attractor region in Table 2, cf. Ref. [32, 31]):

$$\phi_{GAR} = -\frac{GM_{GAR}}{c^2 R_{GAR-MW}} = -1 \times 10^{-4}. \quad (5)$$

The significance of ϕ_{GAR} lies in the fact that it is about five orders of magnitude large than ϕ_{\oplus} . Depending on functional form of a possible VEP – or, quantum-induced violation of equivalence principle, qVEP, introduced in Ref. [17] – this could significantly amplify the local observability of ϕ_{GA} , and hence give us a direct observational probe for the local distribution of cosmological matter.

4. Gravitationally modified wave-particle duality: non-commutative space-time, spacetime foam, and its detectability

In the last several years it has become increasingly clear that interplay of gravitational and quantum effects destroys commutativity of various components of the spacetime vector associated with an event, and in addition it modifies the fundamental commutator. In one spatial dimension, a much studied scenario^{||} for this non-commutativity is captured by the modification [34, 35, 36, 37, 38, 39, 40, 41, 42]:

$$[x, p_x] = i\hbar \xrightarrow{grav.} [x, p_x] = i\hbar \left(1 + \epsilon \frac{\lambda_p p_x^2}{\hbar^2} \right), \quad (6)$$

[§]Borrowing a terminology from the neutrino physics, one may call such states to be “flavor states.”

¶A few-line Erratum to Ref. [22] noted “In retrospect, this paper shows that neutrino oscillations provide a flavor oscillation clock *and* this flavor-oscillation clock redshifts as required by the theory of general relativity.”

^{||}Other scenarios may be found in Ref. [33].

where ϵ is of the order of unity (set equal to unity below). This modification of the fundamental commutator, leads to the following representative consequences/observations:

1. The de Broglie wave-particle duality is modified. It can be encoded in modification of de Broglie's fundamental relation [43]:

$$\lambda_{dB} = \frac{h}{p} \xrightarrow{grav.} \lambda = \frac{\bar{\lambda}_P}{\tan^{-1}(\bar{\lambda}_P/\lambda_{dB})}, \quad (7)$$

where $\bar{\lambda}_P$ is the Planck circumference ($= 2\pi\lambda_P$). The gravitationally-modified λ reduces to λ_{dB} for the low energy regime, and saturates to $4\lambda_P$ in the Planck realm. Not only does this saturation suggests that spacetime loses operational meaning at length scales below Planck length, but it also implies that in spacetime symmetries of quantum gravity there exist at least *two*, rather than one, invariant scales. These are c , and λ_P ; or, c and λ'_P ($\lambda'_P = \epsilon'\lambda_P$, with ϵ' of the order of unity). There is already progress in the development for a relativity with two invariants (c and λ_P) [44, 45].

The indicated saturation of λ to λ_P also implies freezing of neutrino oscillations and disappearance of many interference phenomena [43]. These could have important phenomenological consequences – particularly, for the physics of early universe.

2. The gravitationally-modified wave-particle duality is accompanied with an inherent non-locality [46]. Jack Ng [47] has conjectured that this non-locality may be related to the holographic principle [48, 49, 50].

3. The gravitationally-modified fundamental commutator is associated with a space-time foam – the QGP model of a non-commutative spacetime. For frequencies, $f \ll f_P$, Giovanni Amelino-Camelia [51, 52, 53] has argued – or, at the very least has made strong plausibility arguments – that the power spectrum of strain noise ** is *constant* and can be approximated by

$$\rho_h(f) \approx \frac{\lambda_P}{c} = 5.40 \times 10^{-44} \text{ Hz}^{-1}. \quad (8)$$

Interestingly, such a space-time foam induced white noise is within the reach of currently operating, and planned, gravity wave interferometers [51, 52]. In addition, the associated Planck-scale deformations of the dispersions relations are good candidates for solving a host of observational anomalies [54, 55, 56]

4. Superconducting quantum interference devices (SQUIDs), carry superconducting currents with *temperature-tunable* superconducting mass

$$m_s(T) \sim \eta(T) N_a m_c, \quad (9)$$

behaving as one quantum object (under certain circumstances). In Eq. (9), $N_a \approx 6 \times 10^{23} \text{ mole}^{-1}$, $m_c \approx 2 \times 0.9 \times 10^{-27} \text{ gm}$, and $\eta(T)$ encodes fraction of the available

** Definition: Strain, $h = \Delta L/L$, where, ΔL , is the fluctuation, say induced by gravity-waves (or, space-time fluctuations associated with QGP's spacetime foam) in the relevant distance L .

electrons that are in a superconducting Cooper state at temperature, T . Sufficiently below the critical temperature, $\eta(T)$ may approach unity. The temperature-tunable, $m_s(T)$, can easily compete with the Planck mass, m_P . Thus, SQUIDS carry significant potential to probe wave-particle duality near the Planck scale. One of the theoretical and experimental challenges that remains is to devise an experiment that invokes $m_s(T)$ rather than m_e .^{††}

5. Concluding remarks: Can general-relativistic description of physical reality be considered complete?

Having taken this walk through a variety of terrains in IGQR, I return to the original question: Can general-relativistic description of physical reality be considered complete? The answer is an obvious no – however, the precise nature of this departure is yet to be settled.

It is not the flaw of the theory of general relativity (TGR), for it was never formulated with quantum realm in mind. The founders of TGR did not devise gedanken experiments that used quantum test particles, or nor did they envisage intrinsically quantum sources [57]. Furthermore, while non-commutative spacetime was entertained several decades ago [58], it was realized only in recent years that interplay of the gravitational and quantum realms necessarily leads to a non-commutative – as opposed to TGR’s spacetime continuum – spacetime.

It is abundantly clear that both the gravitational and quantum realms suffer changes as one walks from a strictly (“Strictly,” in the sense as of “as close as practically possible,” etc.) gravitational to a quantum realm, and vice versa. Yet, there are circumstances in which quantum realm enters only at the level of a quantum test particle with no classical analog, and gravitational realm is treated in accordance with TGR. In such circumstances, Can general-relativistic description of physical reality be considered complete? The canonical gedanken experiments, and the principle on which TGR is formulated, invariably invoke local equivalence of the effects of gravity, and that of acceleration [59]. However, one – in an entirely gedanken situation – can imagine the M_{GAR} of the Great Attractor region to be distributed in a spherically symmetric manner around our Milky way; and the radius of such a matter distribution can be visualized to be R_{GAR} . This hypothetical (M_{GAR}, R_{GAR-MW}) configuration does not induce any gravitational forces because $\phi_{(M_{GAR}, R_{GAR-MW})} = -1 \times 10^{-4}$ is constant (i.e. is gradient-less inside the hypothetical matter distribution). Yet, it is responsible for inducing gravitational redshift of flavor-oscillation clocks via *non-zero* gravitationally induced *relative* phases between the underlying mass eigenstates of neutrinos. These relative phases, incidentally, are precisely those which redshift the flavor-oscillation clocks. Now, if one was to think away the Milky way (keeping the neutrinos from distant sources streaming), one is left with a region of spacetime of vanishing Riemann curvature – and the gravitational redshift of flavor-oscillation clocks. For a single mass eigenstate these

^{††} I thank J. L. Smith for arranging a discussion of this subject.

phases are global and hence unobservable. But, when *different* mass eigenstates are superimposed these phases become relative and hence observable. So – while purists would not find it difficult to counter this argument via redshift of planetary orbits – we do have a situation where gravitationally induced quantum mechanical phases exist in a situation where the Riemann curvature identically vanishes. That is, a region with vanishing Riemann curvature cannot be considered devoid of gravitational effects, and gravitational fields. While all gravitationally induced forces vanish in such a region of spacetime, relative gravitationally-induced phases in flavor-oscillation clocks do not. The canonical wisdom captured in Synge’s classic on TGR [60], which reads:

The essence of Einstein’s general theory lies in the assumption that gravitation manifests itself in the curvature of Riemannian space-time. If the Riemann tensor R_{ijkl} of the metric ... were to vanish, we would be back in the flat space-time of gravitationless special relativity. In fact, we may write symbolically

$$R_{ijkl} = \text{gravitational field}, \quad (10)$$

is, therefore, challenged. Gravitation resides beyond Riemann curvature in the spacetime metric. Spacetime metric inside our hypothetical region, and one in free fall outside this region, are not identical. This fact can be ascertained observationally by any good astronomer.

The situation becomes even more interesting if one considers the fuzzy-spin gravitino of Ref. [61] and considers its scattering from a gravitating source [62]. In this situation, as in many other, many of the proclaimed statements on the *geometrical* nature of gravitation fall apart because the founding fathers of TGR had not envisioned that the theory resulting from the stated principle contained Lens-Thirring gravitational field. With this intellectual provocation I leave the remaining ponderings for our audiences’ entertainment without further comment.

We began, with the question: Can general-relativistic description of physical reality be considered complete? The answer which seems to emerge is: No, but precise nature of this “no” is yet to be settled. It is there in those arguments, in those ponderings, that the IGQR and QPG offer some of the greatest fun.

May the phases be with you!

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thanks, on behalf of all Indians, to those who came from places far away and beyond boundaries of India. This work is supported by CONACyT (Mexico) Project 32067-E – to CONACyT my thanks.

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It is impossible to do justice to all the works that touch on the subject of this talk. The general references that I have collected here should help the reader delve deeper into the subject. After this talk was given another experiment in IGQR was reported. It explores quantum states of a neutron in Earth's gravity⁶³. Several seminal works somehow did not weave with the flow of thoughts presented in this talk, and yet they are so important to IGQR that we record one of them here as a gesture of thanks to its author for starting it all [64].

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